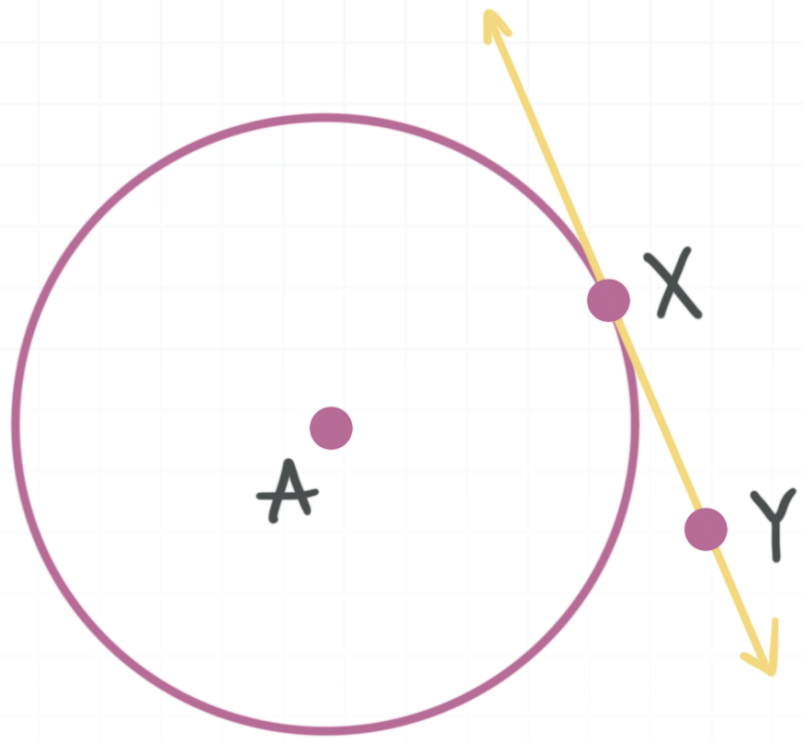
Tangent lines of circles

A **tangent line to a circle** intersects the circle at exactly one point on its circumference.



Line XY is tangent to circle A at point X. Point X is called the **point of tangency**. The radius drawn from the center of the circle to the point of tangency is always perpendicular to the tangent line. In the figure below, Radius $\overline{AX}$ is perpendicular to $\overleftrightarrow{XY}$.

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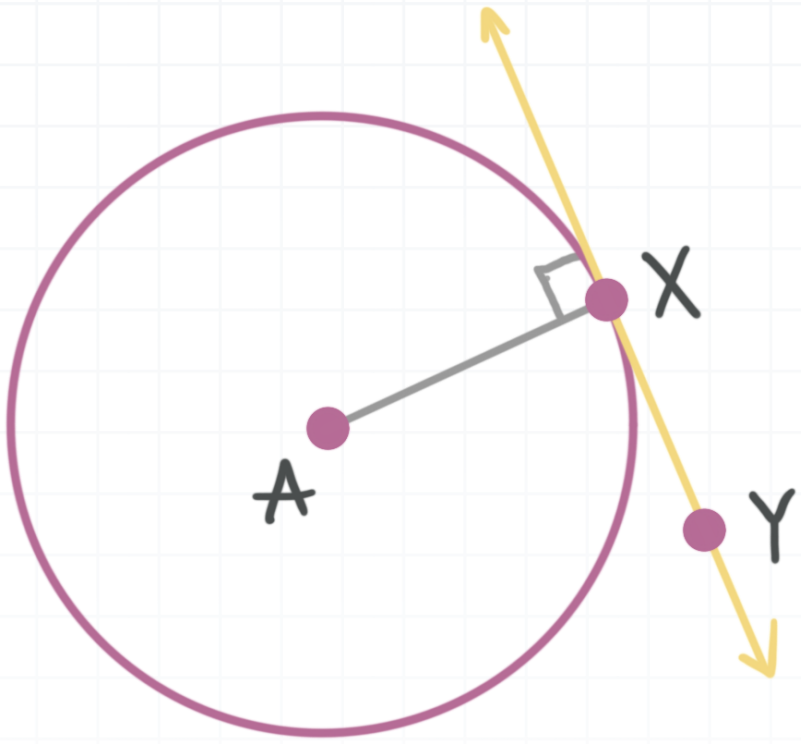
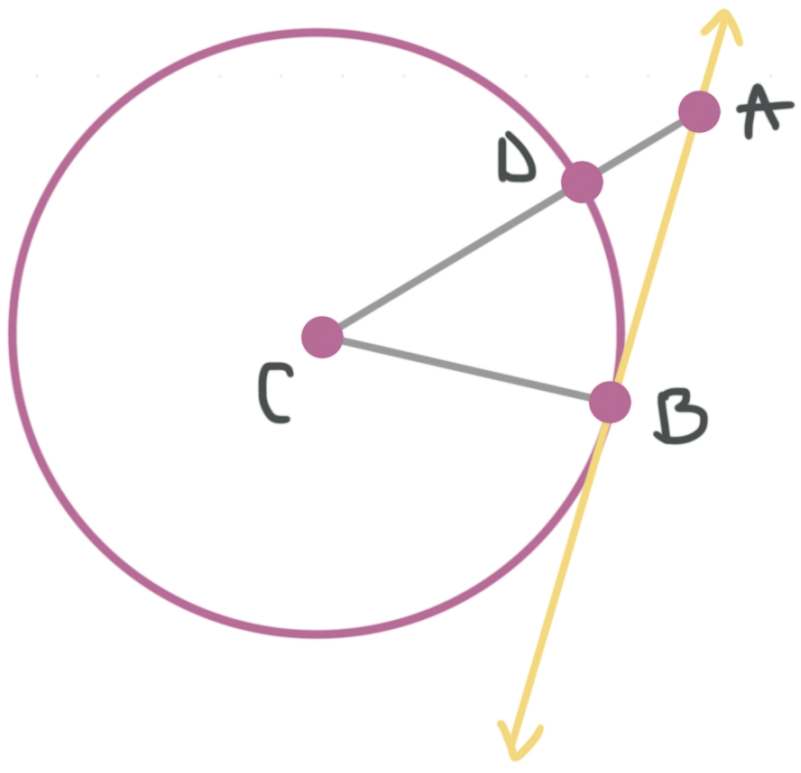


Let’s start by working through an example.

Example

The radius of circle C is 5 units. $AB = 6$ units and $DA = 3$ units. Determine whether line AB is tangent to circle C.



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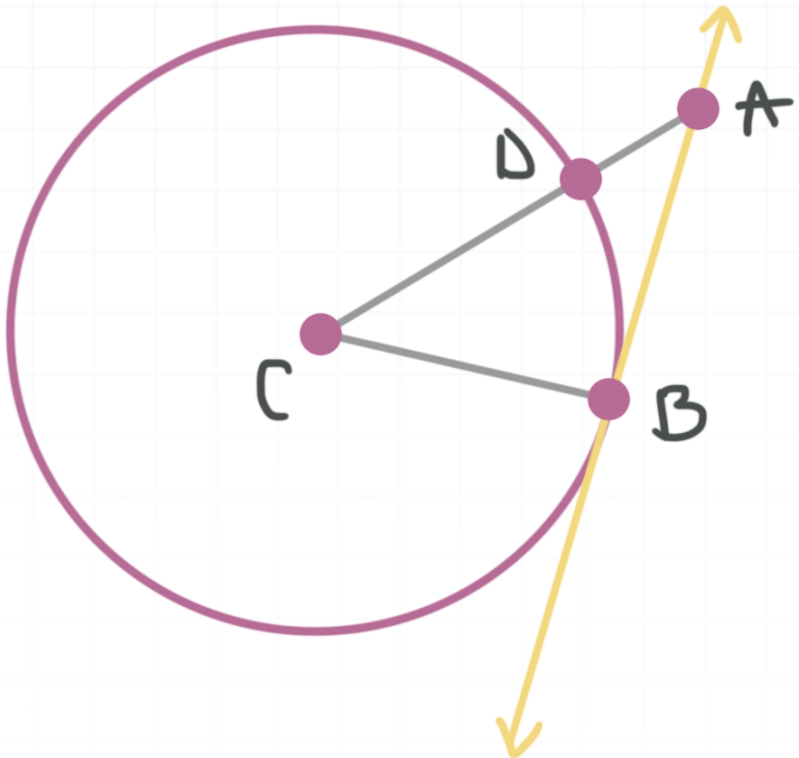
If line AB is tangent to circle C, then the radius will be perpendicular to line AB and angle $\angle CBA$ will be a right angle. If the triangle formed in the diagram is a right triangle, then the Pythagorean theorem will be satisfied for the triangle.



So we want to verify the following equation.

$$CB^2 + AB^2 = CA^2$$

$\overline{CB}$ is a radius so we know $CB = 5$ units, and we know that $AB = 6$ units. We know that $CA = CD + DA$, and that $\overline{CD}$ is also a radius, so $CD = 5$ units. Since $DA = 3$ units, $CA = 5 + 3 = 7$. Now we can check the Pythagorean theorem.

$$CB^2 + AB^2 = CA^2$$

$$5^2 + 6^2 = 7^2$$

$$25 + 36 = 49$$

$$61 \neq 49$$

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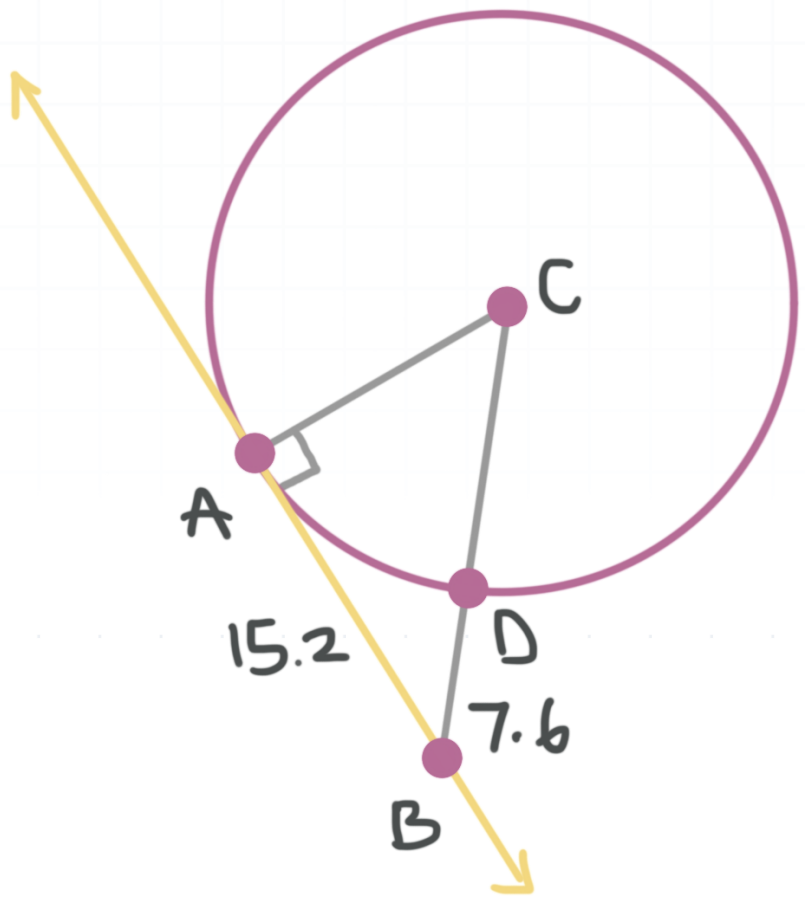
This means that angle $\angle CBA$ is not a right angle, and line AB is therefore not tangent to circle C.

Let's do one more.

Example

Find the length of the radius of circle C, given that $\overleftrightarrow{AB}$ is a tangent line.



A radius drawn to point A will be perpendicular to $\overleftrightarrow{AB}$ and form right triangle BAC.

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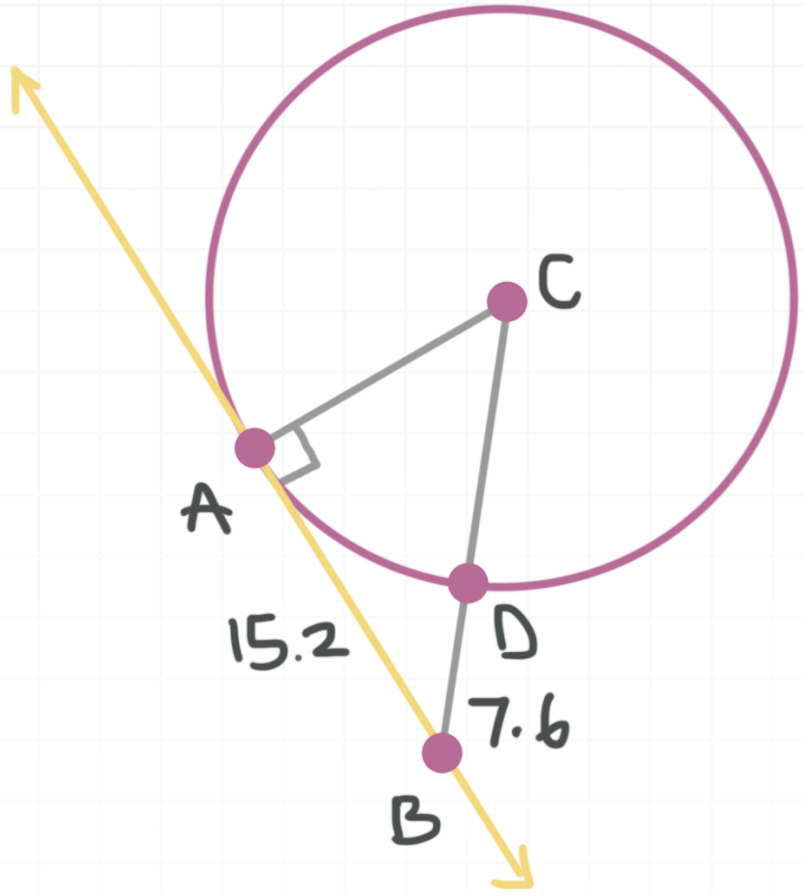

Let's call the length of the radius x, then $AC = x$ and $CD = x$. Now we can use the Pythagorean Theorem to set up an equation and solve for x.

$$x^2 + 15.2^2 = (x + 7.6)^2$$

$$x^2 + 231.04 = x^2 + 15.2x + 57.76$$

Subtract x^2 and 57.76 from both sides and rearrange.

$$15.2x = 173.28$$

$$x = 11.4$$

The radius of circle C is 11.4 units.

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